Trading With a Stock Chart Heuristic

William Leigh, Cheryl J. Frohlich, Steven Hornik, Russell L. Purvis, and Tom L. Roberts

***Abstract*—The efficient market hypothesis (EMH) is a cornerstone of financial economics. The EMH asserts that security prices fully reflect all available information and that the stock market prices securities at their fair values. Therefore, investors cannot consistently "beat the market" because stocks reside in perpetual equilibrium, making research efforts futile. This flies in the face of the conventional nonacademic wisdom that astute analysts can beat the market using technical or fundamental stock analysis. The purpose of this research is to partially assess whether technical analysts, who predict future stock prices by analyzing past stock prices, can consistently achieve a trading return that outperforms the stock market average return. This is tested using knowlege engineering experimentation with one price history pattern—the "bull flag stock chart"—which signals technical analysts of a future stock market price increase. A recognizer for the stock chart pattern is built using a template-matching technique from pattern recognition. The recognizer and associated trading rules are then tested by simulating trading on over 35 years of daily closing price data for the New York Stock Exchange Composite Index. The experiment is then replicated using the horizontal rotation or mirror image pattern of the "bull flag" (or "bear flag" stock chart) that signals a future stock market decrease. Results are systematic, statistically significant, and fail to confirm the null hypothesis based on a corollary to the EMH: that profit realized from trading determined by this heuristic method is no better than what would be realized from trading decisions based on random choice.**

***Index Terms*—Efficient market hypothesis (EMH), expert systems, heuristics, knowledge engineering, stock market forecasting, technical analysis.**

I. Introduction

FOR OVER three decades, a conspicuous contradiction has existed in the financial economic literature. On one side of the contradiction is the efficient market hypothesis (EMH), one of the most widely accepted theories in economics that is a cornerstone of academic finance. Simply stated, the EMH asserts that security prices fully reflect all available information and that the stock market prices securities at their fair values. As a result, the stock market is always perfectly mediated, with no opportunity for above-normal profits, and expenditures on research and trading are a waste of energy and resources. The other side of the contradiction is the conventional (nonacademic) wisdom that astute analysts can beat the market using technical or fundamental stock analysis. Thousands of individuals in the investment management industry spend considerable amounts of time daily, gaining access to information, evaluating that information, and making investment decisions based on that information. Consequently, it has been suggested that, "Either the hypothesis has an inherent flaw, or Wall Street and its customer base are in truth totally irrational" [1].

Charles Dow developed the original theory of technical analysis in 1884. Modern explications of the original Dow theory are periodically published [2], and numerous books and articles that explain and augment the core Dow theory regularly appear in the practitioner literature. Accordingly, a formidable and well-subscribed knowledge base exists on technical analysis stock market forecasting. The purpose of this research is to assess whether returns from trading based on a selected technical analysis market forecasting technique consistently and significantly exceed overall stock market investment return performance. The EMH implies that trading based on this technique will not and cannot outperform overall market performance.

Technical analysts look for patterns in the past price and volume behavior of the stock market to predict future price behavior. This sort of analysis is called "stock charting." The theory is that certain patterns in past behavior precede broad market increases, and certain other patterns in past price and volume behavior precede market decreases. This research comprises knowledge engineering and trading simulation testing with one of the more pronounced price history patterns of the technical analysts—the "bull flag stock chart"—which signals technical analysts of a future stock market price increase. Using a template-matching technique, a pattern recognizer assigns to each trading day in the study scores based on how the immediate time series price history of that trading day conforms to the stock chart pattern. Associated trading rules based on these scores are tested by simulating trading on over 35 years of daily closing price data for the New York Stock Exchange Composite Index. The experiment is then replicated using the horizontal rotation or mirror image pattern of the "bull flag" (or "bear flag" stock chart) that signals a future stock market decrease. Considering the bull flag stock chart pattern from technical analysis to be prior knowledge, i.e., it is not learned or its parameters are optimized, this study may be regarded as confirmatory in nature.

The rest of the paper is organized as follows. Section II defines the theoretical concept of market efficiency and surveys some of the research supporting and challenging it. In the next

Manuscript received February 2, 2002; revised February 27, 2004. This paper was recommended by Associate Editor P. A. Beling.

W. Leigh and S. Hornik are with the Department of Management Information Systems, College of Business, University of Central Florida, Orlando, FL 32816-1400 USA (e-mail: william.leigh@bus.ucf.edu; wleigh@bus.ucf.edu; steven.hornik@bus.ucf.edu).

C. J. Frohlich is with the University of North Florida, Jacksonville, FL 32224-2645 USA (e-mail: cfrohlic@unf.edu).

R. L. Purvis is with the Department of Management, Clemson University, Clemson, SC 29634-5124 USA (e-mail: rlpurvi@clemson.edu).

T. L. Roberts is with the Department of Management and Information Systems, Louisiana Tech University, Ruston, LA 71272 USA (e-mail: troberts@ku.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSMCA.2007.909508

sections, the paper presents the research design and method, reports the results, explores a thought experiment based on the results, speculates about wave motion in the stock market, sums up, and concludes.

II. Efficient EMH and Its Challenges

According to the definition of market efficiency [3], market prices instantly reflect all available information: 1) in the absence of transaction costs; 2) given costless information; and 3) for homogeneous expectations. Right or wrong, most studies assume that these three conditions are met, for practical purposes, in large, modern stock markets. The theory affords three degrees of market efficiency. The strong form of the EMH states that all information (including private information) is immediately factored into the market's price for a security. In this form, no group of investors (particularly those investors with inside or privileged corporate information) has information that allows them to earn abnormal profits. In the semistrong form of the EMH, all public information is considered to be immediately reflected in the price. Most studies on semistrong efficiency have found that the market rapidly adjusts to new public information [4]–[6]. The weak form of the EMH holds that information that can be gained from examining the security's past price and volume trading history is already reflected in the securities price.

Although the EMH has been the basis for much of the research in financial economics [7] and is considered by some to be the best established empirical fact in economics [8], [7], it is not universally accepted. At the extreme, the theory has been called fundamentally flawed [9] and has been dismissed as impossible [10], and it has been argued that stock prices are not rationally related to economic realities, negating any impact of informational efficiency [7]. Other studies challenge certain forms of the EMH. For example, several studies have found that insiders consistently earn abnormal returns on their stock transactions [11]–[14], disputing the strong form of the EMH.

The theory is further challenged by a host of studies that offer soft computing approaches to automate stock trading. These approaches, totally ignoring the EMH, regularly appear in the academic literature and include: 1) neural-network learning from price history [12], [15], [16]; 2) neural network using real-world information [17]; 3) neural network using price history, money supply information, and trading group positions [18]; 4) fuzzy expert system based on several types of information [19]; 5) decision-support system using influence diagram [20]; 6) rough set extraction of trading rules from price history [21]; 7) fuzzy logic engineering [22]; 8) data mining based on signal-processing techniques [23]; and 9) rules [24] or agents [25], combining technical heuristics with other knowledge.

More germane to this research are studies that offer evidence of weak-form market inefficiencies and, therefore, support for technical approaches to market trading and forecasting. These studies, based on market momentum, trend reversal, mean reversion, and investor overreaction, are beginning to appear in prestigious academic journals (e.g., [15] and [26]–[29]). Further, surveys [30], [31] of this growing literature that document "anomalies" to the EMH are now appearing.

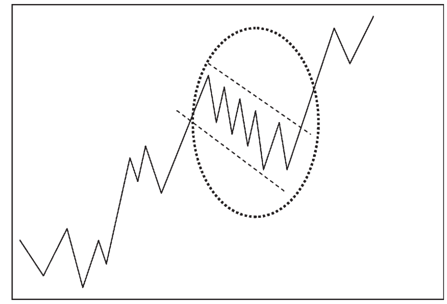


Fig. 1. Representation of a stock price chart showing the "bull flag" stock pattern.

Finally, two studies set out to directly test the use of stock chart heuristics and offer support to the EMH. Fama, an early EMH theorist, tested stock charting against the random-walk model by observing that the price changes, for the period from 1957 to 1962, form a nearly normal frequency distribution (but with one long tail because there were more large increases than small ones) [32]. Fama argues, questionably in our view, that since price changes are normally distributed, stock charting cannot be effective as a stock market trading tool. More recently, Lo *et al.* [33] have used nonparametric kernel regression to search for ten different stock chart patterns (but did not include the bull flag or bear flag patterns used in this paper) in the price histories of individual stocks and failed to find significant profitability in trading simulation experiments, comparing the stock charting decision-making technique to random choice.

III. Research Design and Method

A. Stock Chart Pattern Template Values $T(i, j)$

Fig. 1 illustrates an occurrence of a "flag," a strong type of stock chart pattern found in technical analysis instructional articles and books. A "flag" exemplifies a "consolidation formation," i.e., a period of repetitive price oscillations in a narrow band, such as that marked by the two dashed, parallel, downward-sloping lines in Fig. 1. A bull flag pattern is a horizontal or downward-sloping flag pattern that tracks a price consolidation, followed by a sharp positive price rise termed the "breakout." This occurrence of the bull flag stock chart pattern is encompassed by the dotted oval in Fig. 1. Technical analysts interpret the occurrence of a bull flag pattern as a strong upward price trend being interrupted by a period during which investors "consolidate their gains" before a strong positive bullish breakout occurs and the upward price trend resumes. After the bullish breakout, the price is expected to continue in the upward direction.

Fig. 2(a) contains a quantified synthesis and interpretation of the many examples of occurrences of the bull flag stock chart shown in the technical analysis instructional books, such as [2]. The template T for the bull flag pattern, as shown in Fig. 1, is represented by a 10×10 matrix, whose cells $T(i, j)$ have values that range from -2.5 to $+1.0$. Let the leftmost bottom cell of T, i.e., the cell at row one in column one, be designated as $T(1, 1)$. Cells with no entries have the numerical value

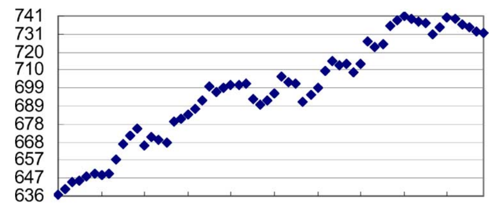
0.5		-1	-1	-1	-1	-1	-1	-1	
1	0.5		-0.5	-1	-1	-1	-1	-0.5	
1	1	0.5		-0.5	-0.5	-0.5	-0.5		0.5
0.5	1	1	0.5		-0.5	-0.5	-0.5		1
	0.5	1	1	0.5				0.5	1
		0.5	1	1	0.5			1	1
-0.5			0.5	1	1	0.5	0.5	1	1
-0.5	-1			0.5	1	1	1	1	
-1	-1	-1	-0.5		0.5	1	1		-2
-1	-1	-1	-1	-0.5		0.5	0.5	-2	-2.5

(a)

-1	-1	-1	-1	-0.5		0.5	0.5	-2	-2.5
-1	-1	-1	-0.5		0.5	1	1		-2
-0.5	-1			0.5	1	1	1	1	
-0.5			0.5	1	1	0.5	0.5	1	1
		0.5	1	1	0.5			1	1
	0.5	1	1	0.5				0.5	1
0.5	1	1	0.5		-0.5	-0.5	-0.5		1
1	1	0.5		-0.5	-0.5	-0.5	-0.5		0.5
1	0.5		-0.5	-1	-1	-1	-1	-0.5	
0.5		-1	-1	-1	-1	-1	-1	-1	

(b)

Fig. 2. Matrix representations for the stock chart pattern templates T used in this paper. The template T for the stock chart patterns is represented with a 10×10 matrix, whose cells $T(i,j)$ have values that range from -2.5 to $+1.0$. Let the leftmost bottom cell of T, i.e., the cell at row one in column one, be designated $T(1,1)$. Cells with no entries have the numerical value of 0.0. (a) Matrix values used for the "bull flag" stock chart pattern. (b) Matrix values for the horizontal rotation or mirror pattern of the bull flag, which is the "bear flag" stock chart pattern.



(a)

0.	0.	0.	0.	0.	0.	0.	0.33	0.83	1.
0.	0.	0.	0.	0.	0.	0.	0.5	0.17	0.
0.	0.	0.	0.	0.	0.	0.5	0.17	0.	0.
0.	0.	0.	0.17	0.5	0.5	0.5	0.	0.	0.
0.	0.	0.	0.5	0.5	0.5	0.	0.	0.	0.
0.	0.	0.33	0.33	0.	0.	0.	0.	0.	0.
0.	0.33	0.33	0.	0.	0.	0.	0.	0.	0.
0.	0.17	0.33	0.	0.	0.	0.	0.	0.	0.
0.33	0.5	0.	0.	0.	0.	0.	0.	0.	0.
0.67	0.	0.	0.	0.	0.	0.	0.	0.	0.

(b)

Fig. 3. Exhibits for a sample computation of $\text{Fit}_{\text{August 5, 1980}}$ and $\text{Height}_{\text{August 5, 1980}}$ for the 60-trading-day price history image window that ends on August 5, 1980. In this paper, 10×10 price history images I_k are constructed to end on each trading day k of the 9000 daily closing prices. This is done in correspondence with a price history image window W_k that, in the first phase of this experiment, is 60 trading days long. A price history image window W_k of trading days is constructed for each trading day k, and a price history image I_k is computed from those closing prices, as described in the text. To complete the computation of the measure of agreement between T and I_k, the Fit_k for the price history image window W_k is calculated by summing the product of the cross-multiplication of the weights $T(i,j)$ in the pattern template with the $I_k(i,j)$ values. The price range, which is the difference from the highest to the lowest prices in a 60-day window ending on trading day k, is divided by the closing price on trading day k to compute a scaled window height for each W_k. (a) Windsorized p_k values for $W_{\text{August 5, 1980}}$. (b) $I_{\text{August 5, 1980}}$.

of 0.0. Fig. 2(b) contains the corresponding horizontal rotation or mirror pattern—the bear flag.

This pattern of positive and negative weighting in Fig. 2(a), and mirrored in Fig. 2(b), is regular and defines areas in the template for the descending price consolidation (leftmost seven columns) and for the upward-tilting price breakout (rightmost three columns) portions of this interpretation of the bull flag heuristic pattern. The columns of the weights in the matrix are equally weighted (the sum of each row's weight in a column equals zero). The weights are selected so that those prices that deviate the farthest from the bull flag pattern (outlined in shading in Fig. 1) will have the largest negative effect on the total fitting score.

This study's first assignment of weights $T(i,j)$, representing the bull flag stock chart pattern, used only 0 and 1 values. Although results with the 0–1 weights were encouraging, a second assignment using fractional weights was devised and tested, as shown in Fig. 3(a). This fractional weighted model was used in this research. No other reassignment of weights, optimization, learning, or data mining was used to improve performance. This, however, is certainly an area of interest for future research.

B. Price History Image Values $I_k(i,j)$

In this paper, 10×10 price history images I_k are constructed to end on each trading day k of the 9000 daily closing prices. This is done in correspondence with a price history image window W_k that, in the first phase of this experiment, is 60 trading days long. The leftmost trading day d_{k-59} included in the 60-day window W_k is the trading day that precedes k, which is the current day of interest by 59 trading days, and the rightmost trading day in the window W_k is trading day k. Thus, 60 daily closing price values $p_{k-59}, \ldots, p_k$ correspond to the 60 trading days in each window W_k.

Within each 60-day window of closing price data, the price time series information is "windsorized" [34] to remove noise by replacing closing price value observations beyond two standard deviations from the mean (of the prices for trading days included in W_k) with the respective two standard deviation (of the prices for trading days in W_k) boundary value. The windsorized closing price values, i.e., p_{k-59} to p_k, for the 60 trading days in a window W_k of history are broken into ten groups of six sequential trading-day closing prices, with each column in the price history image matrix I_k assigned to one of the six-day groups. Let $I_k(1,1)$ designate the cell in the first row and the first column of the 10×10 price history image matrix I_k, i.e., the cell in the lower left-hand corner of I_k. Then, for example, prices $p_{k-59}, \ldots, p_{k-54}$ are mapped to the first column $I_k(i,1)$, and prices $p_{k-5}, \ldots, p_k$ are mapped to the last rightmost column $I_k(i,10)$.

The value of a closing price is used to associate trading-day information to a row within the assigned column in I_k. The price range from the highest to the lowest prices is computed for the prices in the 60-day window as

$$\max_k = \text{maximum } (p_{k-59}, \ldots, p_k)$$
$$\min_k = \text{minimum } (p_{k-59}, \ldots, p_k)$$
$$\text{range}_k = \max_k - \min_k .$$

One-tenth of this difference defines the price interval Δ_k, which is mapped to each of the ten rows of the price history image I_k, i.e.,

$$\Delta_k = \text{range}_k / 10.$$

The value calculated for the cell (i, j) is determined by the price values of the six grouped closing prices for column j, in relation to the price range$_k$ for the closing prices in the window W_k. The mapping process associates higher price values with the higher numbered rows of I_k. $I_k(i, j)$ values range from 0.0 to 1.0, depending on the number of data points that are mapped to each cell. Hence, the top cell of the first column of the price history image I_k is computed as shown at the bottom of the page.

In general, values for cells in row i (with i taking values from 1 to 10) and column j (with j taking values from 1 to 10) for a 10 × 10 price history image matrix and for a price history image window width of 60 trading days are assigned

$$I_k(i, j) = \sum_{s=\alpha}^{\beta} \mu_s$$
$$\sum_{i=1}^{10} I_k(i, j) = 1.0$$

where

- s $= k - 59, \ldots, k$ (*index for the 60 price values in* W_k);
- α $= k - 59 + 6 \cdot (j - 1)$ (*the index of the first trading day mapped to column* j *in* I_k);
- β $= k - 60 + 6 \cdot j$ (*the index of the last trading day mapped to column* j *in* I_k)

and the values of μ_s are given as shown at the bottom of the page.

The resulting price history image $I_k(i, j)$ values range from 0.0 to 1.0, depending on the number of trading-day closing prices that are mapped to each cell. For example, if the trading-day closing prices ranged from \$50 to \$100 over the 60-day window W_k, then the topmost row of I_k would receive the percentage of the six trading days (which is one-tenth of the 60 days in the price history image window W_k) in the respective column, which had a price value in the interval of \$95 to \$100 (which is the highest one-tenth of the price range present in the 60 trading days of the price history image window W_k), and the bottom row cells would receive the percentage of the six trading days in the respective column, which had a price value in the interval of \$50 to \$55. If two of the six values mapped into a column fall into a single cell in the column, then that cell receives the value of 0.333, and the other cells in the column receive values that sum up to 0.667.

It should be noted that if a price history image window size that is different from 60 days is used, the computation of the price history image I_k is also adapted. For example, for 20 days of history, ten groups of two days are used; for 80 days, ten groups of eight days are used; and so forth. Thus, μ_s computations would not use the fraction 1/6 but 1/(number in group).

C. *Fit$_k$ as Cross-Multiplication of Price History Image I_k With Pattern Template T*

A price history image window W_k of trading days is constructed for each trading day k, and a price history image I_k is computed from those closing prices, as described above. This image I_k of values represents the price behavior in the period comprised of trading day k and its 59 trading days of immediate history. I_k has a 10 × 10 matrix form that is consistent with that of the 10 × 10 pattern template image T.

To complete the computation of the measure of agreement between T and I_k, the Fit$_k$ for the price history image window W_k is calculated by summing the product of the cross-multiplication of the weights $T(i, j)$ in the pattern template with the $I_k(i, j)$ values computed from the 60 trading days of price history ending with trading day k, i.e.,

$$\text{Fit}_k = \sum_{i=1}^{10} \sum_{j=1}^{10} (T(i, j) \cdot I_k(i, j)) .$$

$$I_k(10, 1) = \begin{cases} 1.0, & \text{if all six price values } \{p_{k-59}, \ldots, p_{k-54}\} \text{ are in } (\max_k - \Delta_k, \max_k] \\ 0.834, & \text{if five out of six price values } \{p_{k-59}, \ldots, p_{k-54}\} \text{ are in } (\max_k - \Delta_k, \max_k] \\ 0.0, & \text{if none of the six price values, } \{p_{k-59}, \ldots, p_{k-54}\} \text{ are in } (\max_k - \Delta_k, \max_k] \end{cases}$$

$$\mu_s = \begin{cases} 1/6, & \text{if } p_s \text{ in } [\max_k - 10 \cdot \Delta_k, \max_k - 9 \cdot \Delta_k], \text{ for row } i = 1 \\ 1/6, & \text{if } p_s \text{ in } [\max_k - (11 - i) \cdot \Delta_k, \max_k - (10 - i) \cdot \Delta_k], \text{ for row } i = 2, \ldots, 10 \\ 0, & \text{otherwise} \end{cases}$$

Note that the highest values of Fit_k occur when the price history image I_k is in highest conformance with T. Next, the price range, which is the difference from the highest to the lowest prices in a 60-day window ending on trading day k, is divided by the closing price on trading day k to compute a scaled window height for each W_k as follows:

$$\mathrm{Height}_k = \mathrm{range}_k / p_k.$$

D. Example of Computation for $W_{\mathrm{August}\,5,\,1980}$

Fig. 3(a) contains a time series graph of the windsorized New York Stock Exchange Composite Index closing values for the 60-trading-day period ending on August 5, 1980. Only the first trading-day price out of the 60 prices in the window was affected by the treatment. The first trading day's actual closing price value of 632.1 was windsorized to 636.7. Fig. 3(b) shows the $I_{\mathrm{August}\,5,\,1980}$ values computed for $W_{\mathrm{August}\,5,\,1980}$. The resulting $\mathrm{Fit}_{\mathrm{August}\,5,\,1980}$ is -2.92, and the resulting $\mathrm{Height}_{\mathrm{August}\,5,\,1980}$ value is (741.2 − 636.7)/731.0 or 0.14.

E. Experimental Methodology

The stock chart pattern heuristics were treated as prior knowledge; template-matching techniques were used from pattern recognition to implement a recognizer for the stock chart patterns. The developed model is then tested as a classifier to predict an increasing (bull flag) or decreasing (bear flag) future stock price. The values output by the recognizer, i.e., Fit_k and Height_k, are then used to parameterize "trading rules" that identify trading days on which a buy (or sell, in the case of the bear flag) trade is appropriate.

This study is an example of knowledge engineering. General rules (in this case, trading rules based on stock chart recognition) are acquired through the study of published literature. Then, the rules are operationalized with algorithms and computing mechanisms and experimentally tested against data for which results are known. Expert system and knowledge base evaluation may be difficult and often involves many dimensions, but the evaluation of the expert system described in this paper is more structured and less complex than most, as stock trading performance may be defined in a quantified and objective manner. The expert task here, i.e., stock market trading, is amenable to objective and quantitative evaluation through the use of profitability.

The overall average profitability for buying every day of the period of interest and holding for h trading days is then computed. This overall average profitability is then compared to the average profitability attained by buying only on those days identified as buy days by the stock chart trading rules and holding for h trading days. Using the central limit theory, the average profitability of buying on every day in the period and holding for h trading days is equivalent to the average profitability of buying randomly and holding for h days. Therefore, any profit resulting from the application of a trading rule filter that is greater than the average profit for the interval is an "excess" profit. In this way, the EMH is used as the null hypothesis in this paper, and the finding of consistent and statistically significant excess profits constitutes a failure to confirm the EMH.

The Market Average Profit is calculated for both buying strategies for a comparison period as follows. First, define as before

- k — index for the 9000 trading days in the period of the study;
- p_k — closing price value on trading day k;
- Fit_k — fit value computed as described above for trading day k;
- Height_k — height value computed as described above for trading day k.

Then, let

- h — number of trading days in the forecast horizon or holding period for stocks purchased; h is set to 5, 10, 20, 40, 60, 80, 100 days;
- m — index of the first trading day in a period of comparison;
- n — index of the last trading day in a period of comparison;
- σ_{fit} — trading rule parameter value for the Fit_k value;
- σ_{height} — trading rule parameter value for the Height_k value.

The Market Average Profit for a comparison period of $n - m + 1$ trading days for which the trading practice is to buy every day at price p_k and hold each purchase for h trading days before selling at price p_{k+h} now becomes

$$\text{Market Average Profit}(m,n,h) = \sum_{k=m}^{n} \left[(p_{k+h} - p_k)/p_k\right] / (n - m + 1).$$

Next, results are calculated for the same comparison period but, this time, using the following Boolean buying trading rule:

$$\text{Rule } (k, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}) = \text{“If } \mathrm{Fit}_k > \sigma_{\mathrm{fit}} \text{ and } \mathrm{Height}_k > \sigma_{\mathrm{height}} \text{ then buy and hold for } h \text{ trading days and sell.”}$$

First, calculate the number of days a buy decision is made as follows:

$$\text{Number of Buys } (m, n, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}) = \sum_{k=m}^{n} \mathrm{R}_{k,\sigma_{\mathrm{fit}},\sigma_{\mathrm{height}}}$$

where

$$R_{k,\sigma_{\mathrm{fit}},\sigma_{\mathrm{height}}} = \begin{cases} 1, & \text{if Rule } (k, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}) \text{ is true} \\ 0, & \text{otherwise.} \end{cases}$$

Thus

$$\text{Trading Rule Average Profit } (m, n, h, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}) = \sum_{k=m}^{n} \left[((p_{k+h} - p_k) R_k, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}) / p_k\right] / \text{Number of Buys } (m, n, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}}).$$

TABLE I

EXPERIMENTAL RESULTS FOR SIMULATED TRADING WITH A 60-TRADING-DAY PRICE HISTORY IMAGE WINDOW FOR THE STOCK CHART PATTERN TEMPLATES SHOWN IN FIG. 2. THE TRADING RULE TESTED IS, "IF $\mathrm{Fit}_k > \sigma_{\mathrm{fit}}$ AND $\mathrm{Height}_k > \sigma_{\mathrm{height}}$, THEN BUY AND HOLD FOR 60 TRADING DAYS AND SELL." THE FIRST ROW OF RESULTS IS AN OVERALL SUMMARY FOR THE COMPLETE TEST PERIOD OF 9000 TRADING DAYS. THE REMAINING NINE ROWS OF SUMMARY DATA ARE FOR NINE SUBPERIODS OF 1000 TRADING DAYS EACH. (a) BULL FLAG. (b) BEAR FLAG

Period		Market Average Profit %	$\mathrm{Height}_k > 0$					$\mathrm{Height}_k > 0.1$				
			Trading Rule Average Profit % for $\mathrm{Fit}_k >$									
			0	1	2	3	4	0	1	2	3	4
		1.83	1.56	1.68	1.55	1.75	2.27	3.17	3.58	3.49	3.91	4.59
08/04/67	05/12/03	**P-value:**	0.0361	0.1969	0.0826	0.3820	0.0834	0.0000	0.0000	0.0000	0.0000	0.0000
		Number of Buys:	3875	2934	2115	1444	838	1152	950	761	575	382
08/04/67	09/01/71	**0.4**	-0.6	-0.7	-0.9	-0.7	0.9	4.7	5.1	5.2	5.4	6.1
09/02/71	08/19/75	**-0.4**	-1.6	-1.8	-2.0	-0.6	0.8	-1.5	-1.6	-2.1	-1.5	0.3
08/20/75	08/03/79	**1.8**	2.0	2.1	1.8	1.8	1.9	4.2	4.3	4.4	4.5	4.5
08/06/79	07/20/83	**3.1**	3.5	4.5	3.7	3.7	3.5	7.0	8.6	8.3	8.2	7.6
07/21/83	07/06/87	**4.0**	4.0	3.9	4.4	4.2	3.3	3.3	4.4	5.7		
07/07/87	06/18/91	**1.4**	2.8	3.6	3.9	4.2	5.4	5.4	6.4	6.8	7.0	7.6
06/19/91	06/01/95	**2.1**	2.0	2.1	2.2	2.5	3.6					
06/02/95	05/18/99	**4.9**	4.9	5.6	6.7	8.1	8.5	11.7	12.4	12.9	13.4	14.3
05/19/99	05/12/03	**-0.9**	-1.0	-0.8	-1.2	-1.5	-1.5	1.1	1.3	0.8	0.1	0.1

(a)

Period		Market Average Profit %	$\mathrm{Height}_k > 0$					$\mathrm{Height}_k > 0.1$				
			Trading Rule Average Profit % for $\mathrm{Fit}_k >$									
			0	1	2	3	4	0	1	2	3	4
		1.826	1.836	1.845	1.892	1.942	1.840	2.225	2.011	2.076	1.675	1.397
08/04/67	05/12/03	**P-value:**	0.4683	0.4468	0.3357	0.2647	0.4763	0.0442	0.2404	0.1969	0.3464	0.2207
		Number of Buys:	5159	4055	3102	2072	1267	1527	1141	861	563	310
08/04/67	09/01/71	**0.4**	0.7	0.8	1.3	1.5	1.4	4.2	4.6	5.9	6.2	7.7
09/02/71	08/19/75	**-0.4**	-0.3	0.4	0.4	2.0	1.6	0.4	2.1	2.1	3.1	2.8
08/20/75	08/03/79	**1.8**	2.0	2.0	2.2	1.5	1.3	1.2	0.6	0.5	-1.3	-2.9
08/06/79	07/20/83	**3.1**	2.6	2.2	2.1	1.4	1.1	4.0	3.1	3.2	2.3	2.3
07/21/83	07/06/87	**4.0**	4.1	4.0	4.3	4.7	6.0	4.3	4.2	4.4	4.4	5.5
07/07/87	06/18/91	**1.4**	0.0	0.0	0.1	1.0	1.2	-1.6	-2.5	-4.7	-8.8	-8.3
06/19/91	06/01/95	**2.1**	2.5	2.5	2.1	1.6	1.0	0.3	0.3			
06/02/95	05/18/99	**4.9**	4.6	4.8	5.0	5.3	5.8	3.2	3.3	3.6	3.6	3.5
05/19/99	05/12/03	**-0.9**	-1.1	-1.5	-2.2	-3.4	-4.2	0.7	-1.2	-2.4	-4.9	-6.4

(b)

Finally, we have the following "excess" profits for the period of comparison:

$$\begin{aligned}&\text{Average Excess Profit } (m, n, h, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}})\\&\quad = \text{Trading Rule Average Profit } (m, n, h, \sigma_{\mathrm{fit}}, \sigma_{\mathrm{height}})\\&\qquad - \text{Market Average Profit } (m, n, h).\end{aligned}$$

IV. RESULTS OF SIMULATED TRADING

The described research design and method was applied to the New York Stock Exchange Composite Index closing price data for the period from August 4, 1967 to May 12, 2003. This period includes 9000 trading days.

Table I(a) includes experimental results for a 60-trading-day price history image window and a 60-trading-day forecast horizon for the bull flag stock chart pattern template shown in Fig. 2(a). The trading rule tested is, "If $\mathrm{Fit}_k > \sigma_{\mathrm{fit}}$ and $\mathrm{Height}_k > \sigma_{\mathrm{height}}$, then buy and hold for 60 trading days and sell." The first line of data in Table I(a) shows a Market Average Profit of 1.83% for the entire period (buy on all 9000 trading days and hold for 60 days.) Then, the next value, reading from left to right in the first line of data in Table I(a), states that, for $\sigma_{\mathrm{fit}} = 0$ and $\sigma_{\mathrm{height}} = 0$, the Trading Rule Average Profit is 1.56%. This value of 1.56%, of course, is less than the 1.83% Market Average Profit, but to continue reading to the right, for $\sigma_{\mathrm{fit}} = 4$ and $\sigma_{\mathrm{height}} = 0$, the Trading Rule Average Profit is 2.27%, with a Number of Buys for that trading rule of 838 (out of the 9000 trading days in the test period), and a p-value of 0.0834. The p-value is from a one-tailed heteroscedastic t-test. (A Kolmogorov–Smirnov test for normality on the 9000 values for profitability at a 60-trading-day horizon resulted in a p-value less than 0.01, indicating that these data are not normally distributed.)

The entries in the rightmost four columns of the first line of summary data in Table I(a) are results for trading rules of the form, "If $\mathrm{Fit}_k > \sigma_{\mathrm{fit}}$ and $\mathrm{Height}_k > \sigma_{\mathrm{height}}$, then buy and hold for 60 trading days and sell," for $\sigma_{\mathrm{height}} = 0.1$. In the rightmost position of the first row, for $\sigma_{\mathrm{fit}} = 4$ and $\sigma_{\mathrm{height}} = 0.1$, the Trading Rule Average Profit is 4.59%, with a Number of Buys of 382 and a p-value of 0.0000. The Trading Rule Average Profit is higher than the Market Average Profit, and the p-value is 0.0000 for all values of σ_{fit} tested when $\sigma_{\mathrm{height}} = 0.1$. Trading Rule Average Profit ranges from 3.17% up to 4.59%. Note that as the values for σ_{fit} or σ_{height} are increased, the Trading Rule Average Profit generally increases.

The remaining nine rows of summary data in Table I(a) are for nine subperiods of 1000 trading days each. Blank positions

TABLE II

EXPERIMENTAL RESULTS SUMMARIZED FOR 9000 DAYS OF SIMULATED TRADING WITH A 60-TRADING-DAY PRICE HISTORY IMAGE WINDOW FOR THE STOCK CHART PATTERN TEMPLATES SHOWN IN FIG. 2. THE TRADING RULE TESTED IS, "IF $\text{Fit}_k > \sigma_{\text{fit}}$ AND $\text{Height}_k > \sigma_{\text{height}}$, THEN BUY AND HOLD FOR 60 TRADING DAYS AND SELL". (a) BULL FLAG. (b) BEAR FLAG

	Average Excess Profit % for Fit_k >				
Height_k >	0	1	2	3	4
.000	-0.27	-0.15	-0.28	-0.07	0.44
.025	-0.27	-0.14	-0.28	-0.07	0.44
.050	-0.19	-0.01	-0.20	0.02	0.64
.075	0.62	0.95	0.69	0.96	1.54
.100	1.35	1.75	1.66	2.09	2.76

(a)

	Average Excess Profit % for Fit_k >				
Height_k >	0	1	2	3	4
.000	0.010	0.019	0.066	0.116	0.014
.025	0.011	0.020	0.066	0.116	0.014
.050	0.010	0.019	0.066	0.116	0.014
.075	0.088	-0.013	-0.004	-0.019	0.015
.100	0.399	0.185	0.250	-0.151	-0.429

(b)

in the table result when the trading rule did not indicate any buys during the subperiod. Results by subperiod are not uniformly supportive of the method, but there is a general repetition of the pattern of increasing σ_{fit} and σ_{height}, resulting in increasing Trading Rule Average Profit.

Table I(b) includes summary data for simulated trading with the stock chart pattern template shown in Fig. 2(b). This "bear flag" is the stock chart pattern that results from the horizontal rotation, or mirror, of the "bull flag" pattern, which was used to obtain the results reported in Table I(a). Table I(b) has the same structure as Table I(a). The results shown in Table I(b), for a 60-trading-day price history image window width and for a 60-trading-day forecast horizon, are neither conclusive nor suggestive substantively or statistically.

Table II(a) shows the sensitivity of Trading Rule Average Profit for the 9000-trading-day period to changes in σ_{height} for the 60-trading-day price history image window and for the 60-trading-day forecast horizon of Table I(a) for the bull flag stock chart. The pattern of increasing Trading Rule Average Profit as σ_{fit} and σ_{height} are increased is clear, particularly for the higher values of σ_{height}. Fig. 4(a) graphically presents the results. Table II(b) shows corresponding data for the bear flag stock chart. The higher values for the parameters σ_{fit} and σ_{height} show the negative values for Average Excess Profit that technical analysis leads us to expect for this chart.

Table III(a) presents a forecast horizon sensitivity analysis for the 60-trading-day price history image window for a σ_{height} value of 0.1 for the bull flag stock chart. To make the Average Excess Profit statistics comparable across different forecast horizons, the values are scaled by the absolute value of the Market Average Profit as follows:

$$\text{Scaled Average Excess Profit} = \text{Average Excess Profit}/|\text{Market Average Profit}|.$$

Fig. 5(a) shows the bull flag analysis results of Table III(a) in graphical form. The maximum values occur at a forecast horizon of about 20 trading days (for the 60-trading-day price history image window). In general, higher values of σ_{fit} correspond to higher Scaled Average Excess Profit results. Table III(b) and Fig. 5(b) have a similar relationship, but for the bear flag analysis results. For the shorter forecast horizons, the results for the bear flag show the negative values for Scaled

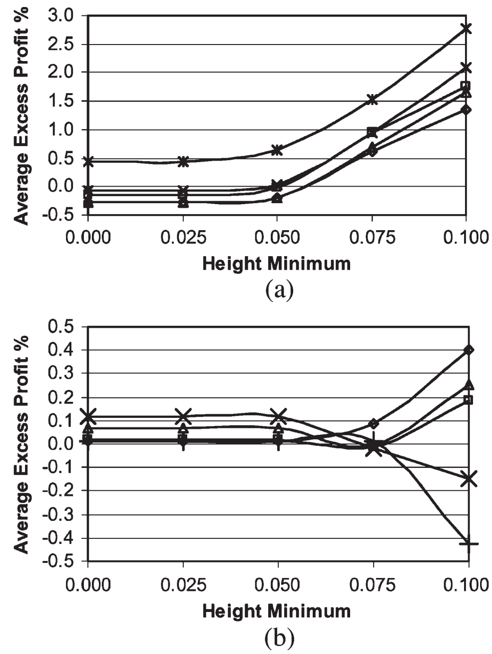


Fig. 4. Each plotted line corresponds to one of the columns of summary data in Table II and shows summary simulated trading results for different fit minimum parameter values in the trading rule. Trading results are sensitive to the height minimum setting in the trading rule for all values of the fit minimum parameter. (a) Bull flag. (b) Bear flag.

TABLE III

EXPERIMENTAL RESULTS SUMMARIZED FOR 9000 DAYS OF SIMULATED TRADING WITH A 60-TRADING-DAY PRICE HISTORY IMAGE WINDOW FOR THE STOCK CHART PATTERN TEMPLATES SHOWN IN FIG. 2. THE TRADING RULE TESTED IS, "IF $\text{Fit}_k > \sigma_{\text{fit}}$ AND $\text{Height}_k > 0.1$, THEN BUY AND HOLD FOR [FORECAST HORIZON] TRADING DAYS AND SELL." (a) BULL FLAG. (b) BEAR FLAG

Forecast	Scaled Average Excess Profit for Fit_k >				
Horizon	0	1	2	3	4
10	1.71	2.01	1.77	1.53	1.66
20	1.67	1.90	1.86	2.02	2.31
30	1.58	1.71	1.53	1.73	2.03
40	1.48	1.65	1.42	1.53	2.01
50	1.17	1.38	1.28	1.44	1.82
60	0.74	0.96	0.91	1.14	1.51
70	0.61	0.77	0.73	0.96	1.33
80	0.54	0.66	0.60	0.79	1.03
90	0.55	0.64	0.57	0.75	0.96
100	0.57	0.68	0.62	0.80	1.04

(a)

Forecast	Scaled Average Excess Profit for Fit_k >				
Horizon	0	1	2	3	4
10	0.16	-0.01	-0.38	-1.35	-2.74
20	0.07	-0.29	-0.43	-1.01	-2.15
30	0.06	-0.28	-0.43	-0.95	-1.40
40	0.30	0.08	0.02	-0.41	-0.72
50	0.28	0.09	0.05	-0.24	-0.42
60	0.22	0.10	0.14	-0.08	-0.23
70	0.32	0.21	0.32	0.34	0.29
80	0.42	0.26	0.24	0.25	0.23
90	0.32	0.17	0.16	0.20	0.26
100	0.24	0.07	0.09	0.21	0.26

(b)

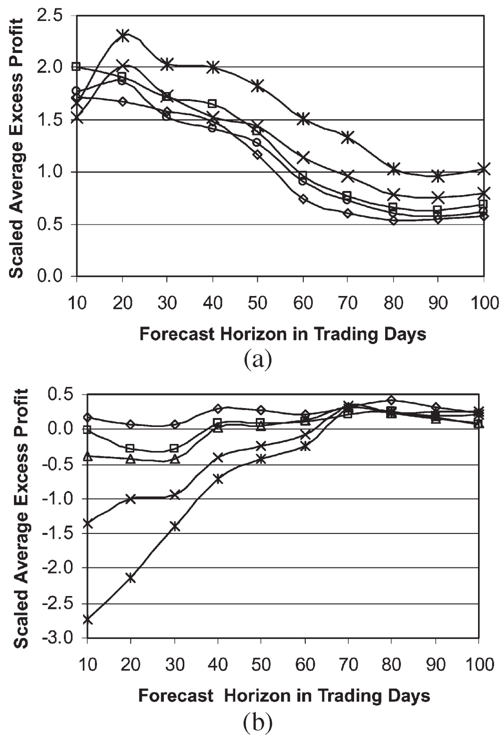


Fig. 5. Each plotted line corresponds to one of the columns of summary data in Table III and shows summary simulated trading results for different fit minimum parameter values in the trading rule. Scaling the average excess profit makes the values comparable for different forecast horizons. Trading results are better for higher minimum fit parameter values and for shorter trading horizons. (a) Bull flag. (b) Bear flag.

Average Excess Profit that technical analysis theory leads us to expect.

In Table III, the best results for the 60-trading-day window occur at about the 20-day forecast horizon. The presentation of results in Table IV is structured in the same way as that in Table I, except for the use of a 20-trading-day forecast horizon (instead of 60). In Table IV(a), the relationship between Trading Rule Average Profit and the parameter values for σ_{fit} and σ_{height} is evident: for increasing parameter values of σ_{fit} and σ_{height}, increasing values for Trading Rule Average Profit and increasing statistical significance are experienced. Statistical significance is strong for the rightmost six columns of Table IV(a). (A Kolmogorov–Smirnov test for normality on the 9000 values for profitability at a 20-trading-day horizon resulted in a p-value less than 0.01, indicating that these data are not normally distributed.)

Table IV(b) shows a profit relationship, opposite that of Table IV(a), for the bear flag experiment between Trading Rule Average Profit and the parameter values σ_{fit} and σ_{height}. The results in Table IV(b) are systematic and achieve statistical significance but are not as strong and clear as the results in Table IV(a). In Table IV(b), for increasing parameter values of σ_{fit} and σ_{height}, decreasing values for Trading Rule Average Profit and increasing statistical significance are experienced. Statistical significance is strong for the rightmost two columns of Table IV(b).

Table V contains the results of a sensitivity analysis on price history image window size. Trading is to occur for those days that have an "above average" value of Fit_k and Height_k, so the trading rule parameters σ_{fit} and σ_{height} are redefined as functions of k, $\sigma_{\text{fit}}(k)$, and $\sigma_{\text{height}}(k)$. The trading rule is, "If $\text{Fit}_k > \sigma_{\text{fit}}(k)$ and $\text{Height}_k > \sigma_{\text{height}}(k)$, then buy and hold for h trading days and sell," where h takes the values of 5, 10, 20, 30, 40, and 50 trading days, and the parameters $\sigma_{\text{fit}}(k)$ and $\sigma_{\text{height}}(k)$ are cumulative averages of Fit_k and Height_k calculated beginning one year (252 trading days) before the beginning of the 9000-trading-day study period and up to and including trading day k. The cumulative average calculation is begun one year before the start of the 9000-trading-day study period so that a stable cumulative average would have been established before the trading rule begins to be applied. Thus, we have

$$\sigma_{\text{fit}}(k) = \sum_{i=-251}^{k} \text{Fit}_i/(k+252)$$

$$\sigma_{\text{height}}(k) = \sum_{i=-251}^{k} \text{Height}_i/(k+252).$$

Fig. 6 is a graphical presentation of the summary results in Table V. Note that as the price history image window is lengthened, the results consistently reflect increases in the respective Scaled Average Excess Profit effects for the bull flag (a), though this does not appear as consistently in the graph for the bear flag (b). Generally, the plots are downward sloping for the bull flag (a) and upward sloping for the bear flag (b).

V. Thought Experiment

The EMH has proven to be a difficult concept to falsify. A large body of very technical literature has been developed in academic finance to defend the EMH from the challenges of the "anomalies." The failure to confirm a null hypothesis, when the hypothesis is so well-established as the EMH, is a strong claim, but not as strong as a claim of falsification of the theory. A claim of falsification requires proof beyond the level of a demonstration of statistical significance.

Our contention is that the experiments reported in this paper are a falsification of the weak form of the EMH. The following is a procedure, a "thought experiment," and an argument that the results in this paper, specifically the results in Table IV(a), constitute this falsification of this special form of the EMH.

- Step 1) Construct a multisided die. Leave as many faces of that die blank as you wish. On the rest of the faces, write instructions of the form, "Buy and hold for h trading days and then sell," with h being any numbers that you wish, which is randomly assigned.
- Step 2) Randomly choose one or several contiguous rows of subperiod data from Table IV(a), but do not include the first row. Now, for the trading days in the subperiod designated for the chosen row or rows, roll the die for each trading day included and, if the die face showing for that roll for that trading day is nonblank, record the amount of the returns from buying on that day and holding for the h days, as instructed by the writing on the die face showing.

TABLE IV

EXPERIMENTAL RESULTS FOR SIMULATED TRADING WITH A 60-TRADING-DAY PRICE HISTORY IMAGE WINDOW FOR THE STOCK CHART PATTERN TEMPLATES SHOWN IN FIG. 2. THE TRADING RULE TESTED IS, "IF $\mathrm{Fit}_k > \sigma_{\mathrm{fit}}$ AND $\mathrm{Height}_k > \sigma_{\mathrm{height}}$, THEN BUY AND HOLD FOR 20 TRADING DAYS AND SELL." THE FIRST ROW OF RESULTS IS AN OVERALL SUMMARY FOR THE COMPLETE TEST PERIOD OF 9000 TRADING DAYS. THE REMAINING NINE ROWS OF SUMMARY DATA ARE FOR NINE SUBPERIODS OF 1000 TRADING DAYS EACH. (a) BULL FLAG. (b) BEAR FLAG

Period		Market Average Profit %	Height$_k$ > 0					Height$_k$ > 0.1				
			Trading Rule Average Profit % for Fit$_k$ >									
			0	1	2	3	4	0	1	2	3	4
08/04/67	05/12/03	0.60	0.67	0.74	0.77	0.87	1.18	1.62	1.76	1.73	1.82	2.00
		P-value:	0.2102	0.0786	0.0647	0.0227	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
		Number of Buys:	3875	2934	2115	1444	838	1152	950	761	575	382
08/04/67	09/01/71	0.2	0.6	0.7	0.5	0.6	0.6	2.1	2.0	1.5	1.4	1.1
09/02/71	08/19/75	-0.3	-0.5	-0.7	-0.7	-0.8	0.0	-0.2	-0.3	-0.5	-0.6	0.2
08/20/75	08/03/79	0.7	1.0	1.0	1.0	1.1	1.9	3.7	3.9	4.0	3.7	3.6
08/06/79	07/20/83	1.0	1.6	2.0	2.1	2.3	2.9	3.2	3.8	4.1	4.3	4.4
07/21/83	07/06/87	1.3	1.2	1.4	2.3	2.9	2.8	3.5	4.0	4.5		
07/07/87	06/18/91	0.5	1.2	1.4	1.6	1.8	2.0	1.6	1.9	2.4	2.5	2.8
06/19/91	06/01/95	0.7	0.7	0.7	0.3	0.2	0.1					
06/02/95	05/18/99	1.6	1.8	2.0	2.9	3.8	4.1	5.8	6.5	6.9	7.3	7.9
05/19/99	05/12/03	-0.3	-0.6	-0.5	-1.0	-1.5	-1.4	0.5	0.6	0.2	-0.1	-0.6

(a)

Period		Market Average Profit %	Height$_k$ > 0					Height$_k$ > 0.1				
			Trading Rule Average Profit % for Fit$_k$ >									
			0	1	2	3	4	0	1	2	3	4
08/04/67	05/12/03	0.60	0.55	0.49	0.47	0.54	0.43	0.65	0.43	0.35	-0.01	-0.69
		P-value:	0.2290	0.0761	0.0783	0.2648	0.1067	0.3685	0.1012	0.0483	0.0007	0.0000
		Number of Buys:	5159	4055	3102	2072	1267	1527	1141	861	563	310
08/04/67	09/01/71	0.2	0.1	0.1	0.3	0.3	0.3	1.3	1.1	1.8	1.8	2.3
09/02/71	08/19/75	-0.3	-0.2	0.0	-0.3	-0.7	-1.4	-0.4	0.1	-0.1	-0.2	-1.3
08/20/75	08/03/79	0.7	0.6	0.4	0.5	0.9	1.5	-1.2	-1.3	-1.4	-2.0	-3.2
08/06/79	07/20/83	1.0	0.5	0.2	0.1	0.0	-0.9	1.1	0.5	0.6	0.5	-1.1
07/21/83	07/06/87	1.3	1.1	1.1	1.4	1.6	2.2	1.0	0.8	0.9	0.6	0.7
07/07/87	06/18/91	0.5	0.1	0.2	0.3	0.6	0.3	1.3	1.8	1.3	-0.8	-1.9
06/19/91	06/01/95	0.7	0.9	1.0	0.8	0.7	0.2	-1.1	-0.4			
06/02/95	05/18/99	1.6	1.5	1.4	1.4	1.4	1.5	-0.1	-0.7	-0.8	-0.7	-0.6
05/19/99	05/12/03	-0.3	-0.3	-0.6	-0.6	-0.8	-0.6	1.5	1.0	0.9	-0.6	-1.4

(b)

TABLE V

EXPERIMENTAL RESULTS FOR SIMULATED TRADING WITH STOCK CHART PATTERN TEMPLATES SHOWN IN FIG. 2 FOR FOUR DIFFERENT PRICE HISTORY IMAGE WINDOW WIDTHS. THE TRADING RULE IS, "IF $\mathrm{Fit}_k > \sigma_{\mathrm{fit}}(k)$ AND $\mathrm{Height}_k > \sigma_{\mathrm{height}}(k)$, THEN BUY AND HOLD FOR h TRADING DAYS AND SELL," WHERE THE PARAMETERS $\sigma_{\mathrm{fit}}(k)$ AND $\sigma_{\mathrm{height}}(k)$ ARE CUMULATIVE AVERAGES OF Fit_k AND Height_k UP TO AND INCLUDING TRADING DAY k. (a) BULL FLAG. (b) BEAR FLAG

Window Width	Number of Buys	Scaled Average Excess Profit					
		Forecast Horizon					
		5	10	20	30	40	50
40	1293	1.18	1.22	1.18	1.21	1.09	0.96
60	1300	1.26	1.41	1.40	1.30	1.19	0.84
80	1232	1.35	1.53	1.59	1.04	0.76	0.46
100	1277	2.21	1.84	0.79	0.39	0.23	0.21

(a)

Window Width	Number of Buys	Scaled Average Excess Profit					
		Forecast Horizon					
		5	10	20	30	40	50
40	1360	-0.08	-0.08	-0.19	-0.29	-0.12	-0.01
60	1278	-0.11	-0.03	-0.26	-0.25	0.13	0.12
80	1423	0.02	-0.43	-0.19	-0.14	0.05	0.14
100	1366	-0.28	-0.06	-0.02	0.15	0.38	0.42

(b)

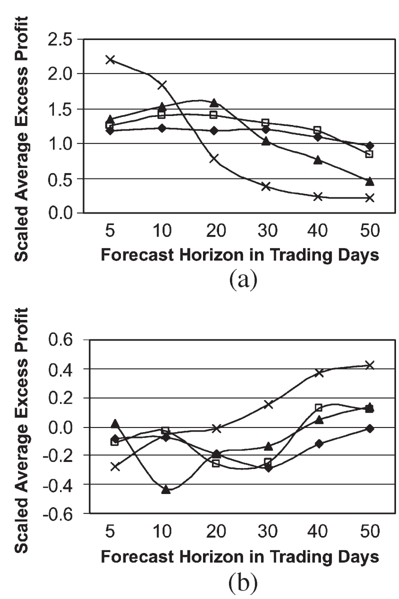


Fig. 6. Each plotted line corresponds to one of the rows of summary data in Table V and shows summary simulated trading results for a different price history image window width. (a) Bull flag. (b) Bear flag.

Sum these returns from this trading by the die for all of the trading days in the subperiod. Call this the "trading-by-die-only-total-return."

Step 3) Randomly choose a row or set of rows from the subperiod summary data in Table IV(a) for a subperiod or group of subperiods that precede the row or rows used for determining the period of the trading-by-die simulation of Step 1). Formulate a trading rule for any cell in this row or rows that have a Trading Rule Average Profit % greater than the Market Average Profit % reported for the subperiod of the row. The trading rule is to be supported by the summary data in the row, specifying buying, if the Fit_k and Height_k of a 60-day price history image window meet the minimum criteria of that cell, included in the headings of Table IV, and holding for 20 trading days, which is the forecast horizon for Table IV. For example, for the row for subperiod 08/04/67 to 09/01/71, a trading rule for the rightmost cell would be, "If the Fit_k is greater than 4.0 and the Height_k is greater than 0.1, then buy and hold for 20 trading days before selling."

Step 4) Do the trading-by-die simulation of Step 2) again for the same trading days with the same die as constructed in Step 1) and used in Step 2). This time, however, if the die face showing for a trading day says, "Buy and hold for 20 days before selling," do not follow this instruction but, instead, follow the dictate of the trading rule developed in Step 2), and sum all of the returns that result from following the rule, or the supplemental 20-day rule, on the die face for all of the trading days in the period simulated in Step 2) to compute a "trading-by-die-augmented-by-stock-chart-heuristic-return."

Repeat these four steps many times. The values in Table IV(a) make it apparent that, for a great majority of executions of the four steps of this thought experiment, the "trading-by-die-augmented-by-stock-chart-heuristic-return" computed in Step 4) will exceed the "trading-by-die-only-total-return" computed in Step 2). Of the 90 cells in Table IV(a) that contain Trading Rule Average Profit % values for subperiod rows, 22 do not exceed the respective subperiod Market Average Profit % values. Of those 22, seven do not exceed because the cells are blank, meaning that no trades were indicated by the minimum Fit_k and Height_k, as specified in the associated trading rules. Of those 22, six are in the bottom row for the period of the stock market crash of 1999–2000.

Now, for example, if the row chosen from Table IV(a) in Step 2) is next to the bottom one, for the subperiod 06/02/95, and the multifaced die constructed in Step 1) has at least one face with the rule, "Buy and hold for h trading days and then sell," then any rule construction in Step 3) is very likely to result in a "trading-by-die-augmented-by-stock-chart-heuristic-return" value, which exceeds the "trading-by-die-only-total-return" value.

Note that for every execution of the four steps of the thought experiment, the state of the three EMH assumption conditions (transactions costs, cost of information, and nature of investor expectations) is identical for Steps 2) and 4), except that there are no more transactions in the Step 4) trading simulation run than in Step 2), making lower transaction costs possible for Step 4).

The subperiod selected in Step 2) may be considered to be a "test sample" for the particular replication of the thought experiment, and the subperiod selected in Step 4) may be considered to be a "learning sample" for the parameter settings to be used in the trading rule. The construction of a learning experiment by bootstrapping the combinations of test and learning samples, as proposed for this thought experiment, is rigorous. The selection of the forecast horizon for the building of the alternative trading rule in Step 3) might draw on tables of summary data similar to Table IV(a), but for other window widths and other forecast horizons. Table V(a) indicates that results that support the effectiveness of trading rules based on the bull flag stock chart at several other window widths and forecast horizons. The selection of these alternative rules for Step 3) for single and multiple replications of the thought experiment might also be accomplished in a bootstrap sort of process.

Table IV(a) is a result set for one stock chart pattern, for one price history image window size, and for one forecast horizon. The technical analysts claim a knowledge base of many stock chart patterns and the ability to apply that knowledge base to many price history image window sizes and for many forecast horizons. The thought experiment is extendible to encompass the results of experimental efforts that are comparable to the one reported in this paper, but for other stock chart patterns and price history image window sizes and forecast horizons. Stock chart patterns that signal market declines would be included in Step 3) as a rule of the form, "If the Fit_k is greater than σ_{fit} and the Height_k is greater than σ_{height}, then sell and hold for h trading days before rebuying."

VI. Wave Motion

Fig. 7 is a redrafting of the summary results behavior for the four price history image window widths included in Table V and Fig. 6. Interpolation is used to supply the data points not included in Table V. The time scale zero corresponds to the lowest point of the bull (a) or highest point of the bear (b) flag stock chart template in the fitting window. Note that for the bull flag (a), as the price behavior through time is graphed so that the low points of the price history image fitting windows for the stock charts coincide, the Scaled Average Excess Profit goes through an inflection (a high for the bull flag and a low for the bear flag) at roughly the same point (about 40 trading days from the pattern inflection point) for the different price history image window widths in (a) and (b). In the range of forecast horizons included in this analysis, the Scaled Average Excess Profit for the bull flag (a) never returns to zero—after initial peaking, the effects (of whatever phenomena caused the price behavior to fit well to the bull flag) attenuate but remain above zero. However, for the bear flag (b), in the range of forecast horizons included in this analysis, the Scaled Average Excess Profit achieves a low and then overshoots attenuation to become positive.

It is intriguing that the curves in Fig. 7(a) are so similar to each other, and so are the curves in Fig. 7(b). One would expect the shorter price history image window widths to detect pattern instances that were not detected by the longer price history image window widths, but that does not seem to be the case. If the same pattern instances are detected by all the window widths, then there may be some underlying very dominant wave

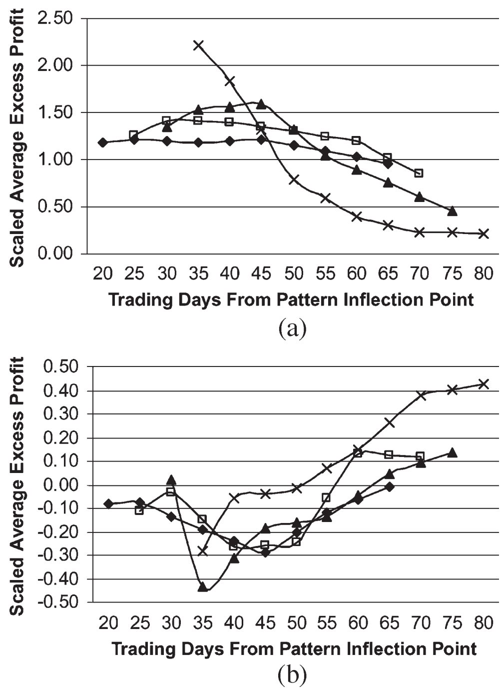


Fig. 7. Each plotted line corresponds to one of the rows of summary data in Table V and shows summary simulated trading results for a different price history image window width. This figure differs from Fig. 6 in that the horizontal axis maps time from the inflection point [(a) low point of the "bull flag" pattern or (b) high point of the "bear flag" pattern] in fitting of the stock chart pattern template used. Plotted points appearing here, which are not included in Table V, are the result of linear interpolation. (a) Bull flag. (b) Bear flag.

length, which is characteristic of the price behavior in the stock markets or in this market index. If the equilibrium of the stock market has physics that resemble the physics of a stretched string, then the bull and bear flags might be interpreted as plucking of that string. This could lead to much speculation and needs further examination.

VII. Summary and Conclusion

This research has used a knowledge engineering approach to evaluate EMH by considering the pronounced stock chart price pattern the "bull flag" that stock market technical analysts consider as a signal for a stock market price increase. The experiment was replicated with the bull flag mirror image pattern: the bear flag that signals a stock market price decrease. The study was implemented by employing a recognizer for this pattern and testing that recognizer and associated heuristic trading rules for multiple forecast horizons and price history image window sizes through 35 years of New York Stock Exchange Index data.

The results are systematic, statistically significant, and generally consistent across subperiods, agree with what the technical analysts tell us to expect, and fail to confirm the null hypothesis: that returns from trading using the predictions of the heuristic trading rules are no better than what would have been realized from trading decisions based on random choice. See the particularly strong results in the rightmost six columns of Table IV(a) and in the rightmost three columns of Table IV(b). A particularly strong result is that the mirror image template pattern of the bull flag template pattern gives profitability results opposite those from the bull flag template pattern. All of these positive results support the technical analysts' claims for the effectiveness of the price pattern heuristic approach (stock charting) to market prediction. The results of this study challenge the weak-form EMH and inspire the consideration of many variations of this experiment, such as other stock chart patterns, price history image window widths, and forecast horizons in future works. The possibilities for further works in learning and optimizing the pattern template are obvious.

The results of this experiment invite speculation as to the underlying stock market mechanisms that may be very effectively described by stock chart pattern heuristics. We searched for recent studies that might suggest the underlying effects at work and found studies of information flow in market processes [35], [36], psychological studies of herd behavior in the stock market [37]–[39], and investor overconfidence [40]. The EMH, as currently formulated, ignores the time effects that can result from information flow and processing mechanisms and should be amended and extended to include these factors.

In future research, we plan to compare the behavior of different markets. Fig. 7 is an example of a response graph that we plan to use as a comparative analytic tool. Examination of this response graph, as prepared for several different stock market indices (such as the Dow Jones Industrial Average), may reveal systematic differences between the respective stock markets and types of securities that comprise the indices.

Although the effectiveness of technical analysis heuristics is difficult to explain with currently accepted academic theory, we think that further analysis of stock market dynamic behavior as wave motion (characteristic of energy systems and/or of information diffusion systems), as suggested in Fig. 7, could lead to an integrated understanding of the stock market that is superior to the "random walk" model implied by the EMH. Practical thinking and experimentation such as this can be and should be a precursor to the development of a formal theory. In due course, normal science will develop models to explain these and all of the other EMH anomalies, which are currently studied as isolated phenomena, in an integrated way.

References

[1] P. L. Bernstein, "A new look at the efficient market hypothesis," *J. Portf. Manage.*, vol. 25, no. 2, pp. 1–2, Winter 1999.
[2] R. Edwards and J. Magee, *Technical Analysis of Stock Trends*. New York: Amacom, 1997.
[3] E. Fama, "Efficient capital markets: A review of theory and empirical work," *J. Finance*, vol. 25, no. 2, pp. 383–417, May 1970.
[4] N. Chopra, J. Lakonishok and J. Ritter."Measuring abnormal performance do stocks overact?" *J. Financ. Econ.*, vol. 31, pp. 235–268, 1992.
[5] E. Fama, L. Fisher, M. Jensen, and R. Roll, "The adjustment of stock prices to new information," *Int. Econom. Rev.*, vol. 10, no. 1, pp. 2–21, Feb. 1969.
[6] R. Ibbotson, J. Sindelar, and J. Ritter, "Initial public offerings," *J. Appl. Corp. Finance*, vol. 1, pp. 37–45, Summer 1988.
[7] L. Summers, "Does the stock market rationally reflect fundamental values?" *J. Finance*, vol. 41, no. 41 pp.591–601, Jul. 1986.
[8] M. Jensen, "Symposium on some anomalous evidence regarding market efficiency," *J. Financ. Econ.*, vol. 6, pp. 93–330, 1978.

[9] R. Ball, “The development, accomplishments and limitations of the theory of stock market efficiency,” *Manage. Finance*, vol. 20, no. 2/3, pp. 3–48, 1994.
[10] S. Grossman and J. Stiglitz, “On the impossibility of informationally efficient markets,” *Amer. Econ. Rev.*, vol. 70, no. 3, pp. 393–408, Jun. 1980.
[11] J. Jaffe, “Special information and insider trading,” *J. Bus.*, vol. 47, no. 3, pp. 410–428, Jul. 1974.
[12] K. Nunn, G. Madden and M. Gombola, “Are some investors more ‘inside’ than others?” *J. Portf. Manage.*, vol. 9, pp. 19–22, Spring 1983.
[13] M. Rozeff and M. Zaman, “Market efficiency and insider trading: New evidence,” *J. Bus.*, vol. 61, no. 1, pp. 25–45, Jan. 1988.
[14] E. Fama, “Efficient capital markets,” *J. Finance*, vol. 46, no. 5, pp. 1575–1617, Dec. 1991.
[15] R. Gencay, “Optimization of technical trading strategies and the profitability in security markets,” *Econ. Lett.*, vol. 59, no. 2, pp. 249–254, May 1998.
[16] A. Refenes, A. Zapranis, and G. Francis, “Stock performance modeling using neural networks: A comparative study with regression models,” *Neural Netw.*, vol. 7, no. 2, pp. 375–388, 1994.
[17] K. Kohara, T. Ishikawa, Y. Fukuhara, and Y. Nakamura, “Stock price prediction using prior knowledge and neural networks,” *Intell. Syst. Accounting, Finance Manage.*, vol. 6, pp. 11–22, 1997.
[18] G. Grudnitski and L. Osburn, “Forecasting S and P and gold futures prices: An application of neural networks,” *J. Futures Mark.*, vol. 13, no. 6, pp. 631–643, 1993.
[19] K. Lee and W. Kim, “Integration of human knowledge and machine knowledge by using fuzzy post adjustment: Its performance in stock market timing prediction,” *Expert Syst.*, vol. 12, no. 4, pp. 331–338, 1995.
[20] K. Poh, “An intelligent decision support system for investment analysis,” *Knowl. Inf. Syst.*, vol. 2, no. 3, pp. 340–358, Aug. 2000.
[21] K. Kim and I. Han, “The extraction of trading rules from stock market data using rough sets,” *Expert Syst.*, vol. 18, no. 4, pp. 194–202, Sep. 2001.
[22] H. Dourra and P. Siy, “Stock evaluation using fuzzy logic,” *Int. J. Theor. Appl. Financ.*, vol. 4, no. 4, pp. 585–602, 2001.
[23] M. Last, Y. Klein, and A. Kandel, “Knowledge discovery in time series databases,” *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 31, no. 1, pp. 160–169, Feb. 2001.
[24] J. Armstrong and F. Collopy, “Causal forces: Structuring knowledge for time-series extrapolation,” *J. Forecast.*, vol. 13, pp. 103–115, 1993.
[25] S. Skouras, “Financial returns and efficiency as seen by an artificial technical analyst,” *J. Econ. Dyn. Control*, vol. 25, no. 1/2, pp. 213–244, Jan. 2001.
[26] H. Hong, T. Lim, and J. Stein, “Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies,” *J. Finance*, vol. 55, no. 1, pp. 205–295, Feb. 2000.
[27] H. Hong and J. Stein, “A unified theory of underreaction, momentum trading, and overreaction in asset markets,” *J. Finance*, vol. 54, no. 6, pp. 2143–2184, Dec. 1999.
[28] L. Chan, N. Jegadeesh, and J. Lakonishok, “Momentum strategies,” *J. Finance*, vol. 51, pp. 1681–1713, 1996.
[29] G. Caginalp, D. Porter, and V. Smith, “Momentum and overreaction in experimental asset markets,” *Int. J. Ind. Organ.*, vol. 18, no. 1, pp. 187–204, Jan. 2000.
[30] J. Conrad and G. Kaul, “An anatomy of trading strategies,” *Rev. Financ. Stud.*, vol. 11, no. 3, pp. 489–519, 1998.
[31] G. Frankfurter and E. McGoun, “Anomalies in finance: What are they and what are they good for?” *Int. Rev. Financ. Analy.* vol. 10, no. 4, pp. 407–429, 2001.
[32] E. Fama, “The behavior of stock market prices,” *J. Bus.*, vol. 38, no. 1, pp. 34–105, Jan. 1965.
[33] A. Lo, H. Mamaysky, and J. Wang, “Foundations of technical analysis: Computational algorithms, statistical inference, and empirical implementation,” *J. Finance*, vol. 55, no. 4, pp. 1705–1765, Aug. 2000.
[34] T. Sargent, *Bounded Rationality in Macroeconomics*. Oxford, U.K.: Oxford Univ. Press, 1995.
[35] G. Franke and D. Hess, “Information diffusion in electronic and floor trading,” *J. Empir. Finance*, vol. 7, no. 5, pp. 455–478, Dec. 2000.
[36] T. George and C. Hwang, “Information flow and pricing errors: A unified approach to estimation and testing,” *Rev. Financ. Stud.*, vol. 14, no. 4, pp. 979–1020, Oct. 2001.
[37] L. Nelson, “Persistence and reversal in herd behavior: Theory and application to the decision to go public,” *Rev. Financ. Stud.*, vol. 15, no. 1, pp. 65–95, Mar. 2002.
[38] E. Chang, J. Cheng, and A. Khorana, “An examination of herd behavior in equity markets: An international perspective,” *J. Bank. Finance*, vol. 24, no. 10, pp. 1651–1679, Oct. 2000.
[39] A. Banerjee, “A simple model of herd behavior,” *Q. J. Econ.*, vol. CVII, no. 3, pp. 797–817, Aug. 1992.
[40] K. Daniel and S. Titman, “Market efficiency in an irrational world,” *Financ. Anal. J.*, pp. 28–40, Nov./Dec. 1999.

William Leigh is a Professor of management information systems in the Department of Management Information System, College of Business, University of Central Florida, Orlando. He is the author of several papers published in the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS, *Decision Support Systems*, and other journals. He is currently investigating the use of technical indicators for forecasting the stock market.

Cheryl J. Frohlich received the M.B.A. degree from Illinois State University, Normal, and the master’s and Ph.D. degrees in finance from the University of Illinois, Urbana.

She is with the University of North Florida, Jacksonville, as an Associate Professor of finance. She has worked extensively in the banking industry and continues to do consulting. She is the author of several articles on banking, investing, and management topics published in scholarly journals, conferences, and practitioner publications, such as the *Journal of Service Marketing*, the *Journal of Business Strategy*, *Advances in Financial Planning and Forecasting*, the *Journal of Business and Psychology*, the *Journal of Business Strategies*, the *Journal of Business Finance and Accounting*, the *Journal of Portfolio Management*, the *Journal of Financial Research*, *Review of Quantitative Finance and Accounting*, and *Florida Banking*.

Dr. Frohlich has received several awards for teaching and research, including the University’s Outstanding Undergraduate Teaching Award, Finance Teacher of the Year Award, McKnight Fellowship, Kip Research Fellowship, and Securities and Futures Authority’s Outstanding Paper in Investments.

Steven Hornik is an Assistant Professor of management information systems in the Department of Management Information System, College of Business, University of Central Florida, Orlando. His research interests include the effectiveness of technology-mediated learning for training and higher education and the social implications of technology. He has been involved in the Research Initiative for Teaching Effectiveness, University of Central Florida, to examine the impact of distance learning outcomes. His research has focused on the relationship between learner variables (social presence, confidence, and motivation), the use of technology for learning (self-efficacy), and learning outcomes.

Russell L. Purvis received the B.S. degree from the University of Miami, Coral Gables, FL, the M.B.A. degree from Georgia State University, Atlanta, and the Ph.D. degree in business administration (MIS) from Florida State University, Tallahassee.

He is an Associate Professor in the Department of Management, Clemson University, Clemson, SC. His current research interests include organizational transformation through information technologies, issues in the implementation of IT applications within organizations, project management, and stock market returns based on the technical analysis market forecasting technique. He has had papers accepted for publication in *Decision Support Systems*, the IEEE TRANSACTIONS ON ENGINEERING MANAGEMENT, *Information and Management*, *Management Science, and Organization Science*, among others.

Tom L. Roberts received the M.B.A. and Ph.D. degrees in information systems from Auburn University, Auburn, AL.

He is currently the Clifford R. King Professor of Information Systems with the Department of Management and Information Systems, Louisiana Tech University, Ruston. His work has been published in the *Journal of Management Information Systems*, *Information and Management*, *Journal of the Association of Information Systems*, and others. His research interests include information assurance, collaborative software and technology, information systems personnel issues, and project management.

Dr. Roberts has work published in the IEEE TRANSACTIONS ON SOFTWARE ENGINEERING and the IEEE TRANSACTIONS ON PROFESSIONAL COMMUNICATION.